Review Problems

March 24, 2017

1. (Fall 2006, Exam 3, #6) Each of the following series converge

$$S_1 = \sum_{n=1}^{\infty} (-1)^n \frac{2}{3n+1}, \; S_2 = \sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}, \; S_3 = \sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n\sqrt{n}}.$$

   Which ones converge absolutely?

2. (Fall 2007, Exam 3, #1) True or False: The series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2+1}$ converges absolutely.

3. (Fall 2007, Exam 3, #3) Suppose we know that $a_n \le \frac{1}{2n}$, $n = 1, 2, 3, \ldots$. Which statement below must be true?

   A) $\sum_{n=1}^{\infty} a_n$ converges

   B) $\sum_{n=1}^{\infty} a_n$ diverges

   C) $\sum_{n=1}^{\infty} a_n$ converges, provided $\lim_{n\to\infty} a_n = 0$

   D) $\sum_{n=1}^{\infty} a_n$ converges, provided $a_n \ge a_{n+1}$ for all n

   E) None of the statements above is necessarily true

4. (Fall 2007, Exam 3, #5) Is the following series convergent or divergent, and why?

$$\sum_{k=1}^{\infty} \frac{5^k k^k}{(2k-1)^{2k}}$$

5. (Fall 2007, Exam 3, #6) Is the following series convergent or divergent, and why?

$$\sum_{m=1}^{\infty} \frac{m!}{4^{2m} m^4}$$

6. (Fall 2007, Exam 3, #7) Which of the following series converge?

$$I) \sum_{k=1}^{\infty} \frac{1}{k(\ln k)^2} ; II) \sum_{k=2}^{\infty} \frac{1}{k(\ln k)^{1/3}}$$